Instructions: Try these problems from the textbook as practice (answers below).

Problems

1-7. Find the cross product of $\mathbf{a} \times \mathbf{b}$ and verify that it is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$.

1. $\mathbf{a} = \langle 2, 3, 0 \rangle, \mathbf{b} = \langle 1, 0, 5 \rangle$
3. $\mathbf{a} = 2\mathbf{j} - 4\mathbf{k}, \mathbf{b} = -\mathbf{i} + 3\mathbf{j} + \mathbf{k}$
5. $\mathbf{a} = \frac{1}{2}\mathbf{i} + \frac{1}{3}\mathbf{j} + \frac{1}{4}\mathbf{k}, \mathbf{b} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$
7. $\mathbf{a} = \langle t, 1, 1/t \rangle, \mathbf{b} = \langle t^2, t^2, 1 \rangle$

9-12. Find the vector, not with determinants, but by using properties of cross products.

9. $(\mathbf{i} \times \mathbf{j}) \times \mathbf{k}$
11. $(\mathbf{j} - \mathbf{k}) \times (\mathbf{k} - \mathbf{i})$

17. If $\mathbf{a} = \langle 2, -1, 3 \rangle$ and $\mathbf{b} = \langle 4, 2, 1 \rangle$, find $\mathbf{a} \times \mathbf{b}$ and $\mathbf{b} \times \mathbf{a}$.

19. Find two unit vectors orthogonal to both $\langle 3, 2, 1 \rangle$ and $\langle -1, 1, 0 \rangle$.

21. Show that $\mathbf{0} \times \mathbf{a} = \mathbf{0} = \mathbf{a} \times \mathbf{0}$ for any vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$.

27. Find the area of the parallelogram with vertices $A(-3, 0), B(-1, 3), C(5, 2)$ and $D(3, -1)$.

29-32. Find a nonzero vector orthogonal to the plane through the points P, Q, R and find the area of the triangle PQR.

29. $P(1, 0, 1), Q(-2, 1, 3), R(4, 2, 5)$
31. $P(0, -2, 0), Q(4, 1, -2), R(5, 3, 1)$

33. Find the volume of the parallelepiped determined by the vectors $\mathbf{a} = \langle 1, 2, 3 \rangle$, $\mathbf{b} = \langle -1, 1, 2 \rangle$ and $\mathbf{c} = \langle 2, 1, 4 \rangle$.

35. Find the volume of the parallelepiped with adjacent edges PQ, PR, PS where $P(-2, 1, 0)$, $Q(2, 3, 2)$, $R(1, 4, -1)$, $S(3, 6, 1)$.

37. Use the scalar triple product to verify that the vectors $\mathbf{u} = \mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$ and $\mathbf{w} = 5\mathbf{i} + 9\mathbf{j} - 4\mathbf{k}$ are coplanar.

43. If $\mathbf{a} \cdot \mathbf{b} = \sqrt{3}$ and $\mathbf{a} \times \mathbf{b} = \langle 1, 2, 2 \rangle$, find the angle between $\mathbf{a}$ and $\mathbf{b}$.

Answers

1. $15\mathbf{i} - 10\mathbf{j} - 3\mathbf{k}$
3. $14\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$
5. $-\frac{3}{2}\mathbf{i} + \frac{7}{4}\mathbf{j} + \frac{2}{3}\mathbf{k}$
7. $(1-t)\mathbf{i} + (t^3 - t^2)\mathbf{k}$
9. $\mathbf{0}$
11. $\mathbf{i} + \mathbf{j} + \mathbf{k}$
17. $\langle -7, 10, 8\rangle$; $-\langle -7, 10, 8\rangle$
19. $\left\langle -\frac{1}{3\sqrt{3}}, -\frac{1}{3\sqrt{3}}, \frac{5}{3\sqrt{3}} \right\rangle$, $\left\langle \frac{1}{3\sqrt{3}}, \frac{1}{3\sqrt{3}}, -\frac{5}{3\sqrt{3}} \right\rangle$
27. 20
29. $\langle 0, 18, -9\rangle$, $\frac{9}{2}\sqrt{5}$
31. $\langle 13, -14, 5\rangle$, $\frac{1}{2}\sqrt{390}$
33. 9
35. 16
43. $60°$